# Math 251 Final Exam (Practice)

**Name**: ____________________

**This exam has 12 questions, for a total of 120 points.**

Please answer each question in the space provided. Please write **full solutions**, not just answers. Cross out anything the grader should ignore and circle or box the final answer. As always, watch out for typos and errors. If you notice any, please let me know.

| Question | Points | Score |
|---|---|---|
| 1 | 6 | |
| 2 | 8 | |
| 3 | 10 | |
| 4 | 8 | |
| 5 | 12 | |
| 6 | 10 | |
| 7 | 12 | |
| 8 | 10 | |
| 9 | 12 | |
| 10 | 8 | |
| 11 | 12 | |
| 12 | 12 | |
| Total: | 120 | |

**Question 1. (6 pts)**

Find all possible values of $a$ so that the plane

$$ax + y = 1$$

forms a angle of 45 degrees with the line

$$\frac{x-1}{2} = \frac{y}{2} = z - 1$$

**Question 2. (8 pts)**

Determine whether

$$\lim_{(x,y)\to(0,0)} \frac{x^2 + \sin^2 y}{4x^2 + 3y^2}$$

exists.

**Question 3. (10 pts)**

Given

$$f(x,y) = x^2 + \sin(xy)$$

(a) Find the directional derivative of $f(x,y)$ in the direction $\langle 1, -1 \rangle$ at the point $(1, \pi)$;

(b) Find the tangent plane to the graph of $f(x,y)$ at the point $(1, \pi, 1)$.

**Question 4. (8 pts)**

Use differentials to approximate $\sqrt{0.96} \cdot e^{0.01}$.

**Question 5. (12 pts)**

For this question, choose one (and only one) of the following two versions.

**(Version A)** Find the local maximum, minimum and saddle points of

$$f(x, y) = x^3 + y^3 - 3x^2 - 12y$$

**(Version B)** Find the absolute maximum and minimum values of

$$f(x, y) = x^2y + 2x^2 + y^2$$

on $x^2 + 2y^2 = 12$.

**I choose version (circle one) A. B.**

**Question 6. (10 pts)**

Given the triple integral

$$\iiint_E (x^2 + z^2)\ dV$$

where $E$ is the part of the unit ball in the first octant

(a) write the integral in $xyz$ coordinates.

(b) write the integral in spherical coordinates.

**Question 7. (12 pts)**

(a) Determine if

$$\mathbf{F}(x,y,z) = \langle 2x + e^x z,\ \sin y,\ e^x \rangle$$

is a conservative vector field. If it is, find a function $f$ such that $\nabla f = \mathbf{F}$.

(b) Evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where $C$ is the curve

$$\mathbf{r}(t) = \cos(\pi t)\mathbf{i} + \sin(\pi t)\mathbf{j} + t^2\mathbf{k}, \quad 0 \le t \le 1$$

**Question 8. (10 pts)**

Evaluate

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$

where $\mathbf{F}(x, y, z) = \langle y,\, x,\, z \rangle$ and $S$ is the part of the paraboloid $z = 2 - x^2 - y^2$ above the plane $z = 1$. Assume $S$ is oriented downward.

**Question 9. (12 pts)**

Evaluate

$$\oint_C xy\ dx$$

where $C$ is the closed curve that consists of the upper half of the unit circle $x^2 + y^2 = 1$ and the part of the parabola $y = x^2 - 1$ below the $x$-axis. Assume $C$ is oriented counterclockwise.

**Question 10. (8 pts)**

Evaluate

$$\int_0^1 \int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} \cos(x^2+y^2)dydx + \int_1^2 \int_0^{\sqrt{4-x^2}} \cos(x^2+y^2)dydx.$$

**Hint: you need to rewrite the integral.**

**Question 11. (12 pts)**

Evaluate

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$

where $\mathbf{F} = \langle 2e^x y, -e^x y^2 + y, z + \cos x \rangle$ and $S$ is the surface of the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = -1$ and $x + y + z = 1$. Assume $S$ is oriented outward.

**Question 12. (12 pts)**

Evaluate

$$\iint_S \operatorname{curl}\mathbf{F} \cdot d\mathbf{S}$$

where $\mathbf{F} = e^{xy}\sin z\mathbf{i} + xz^2\mathbf{j} + yz\mathbf{k}$ and $S$ is the hemisphere $x = \sqrt{1-y^2-z^2}$, oriented towards the positive $x$-axis.